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SHORTER COMMUNICATIONS

COMMENTS ON THE EFFECT OF AXIALLY VARYING AND UNSYMMETRICAL BOUNDARY CONDITIONS ON HEAT TRANSFER WITH TURBULENT FLOW BETWEEN PARALLEL PLATES

A. P. HATTON and A. QUARMBY, *Int. J. Heat Mass Transfer* **6**, 903–914 (1963)

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THESE comments are directed toward the application of equations (24) and (25) to correlation of experimental data for parallel plates with axially uniform, but unsymmetrical, heat fluxes. A characteristic heat-transfer coefficient that the independent of heat-flux symmetry is defined as follows:

$$h_s = (q_i - q_o)/t_i + t_o - 2t_m)$$

where the characteristic heat-transfer coefficient is indicated by the suffix s and other symbols correspond to Hatton and Quarmby's nomenclature.

This coefficient is numerically equal to that for symmetrical heat transfer, and is useful for predicting the coefficients for other unsymmetrical situations. It is constant with respect to changing heat-flux symmetry for turbulent flow in so far as the five assumptions made by Hatton and Quarmby are valid.

It has been shown by Madsen and Thompson [1] that the same coefficient is applicable to the laminar regime.

On Fig. 9 of Hatton and Quarmby's paper the effect of unsymmetrical heat fluxes on fully developed Nusselt numbers is shown. If, instead of the ratio $(Nu_\infty)_o/(Nu_\infty)_{o,\ q_i/q_o = 0}$, the reciprocal of this ratio is plotted against the heat-flux ratio the result is a straight line, Fig. 11.

This relationship is useful for correlating experimental data if Nusselt numbers are known for fixed Reynolds and Prandtl numbers but with varying heat-flux ratios.

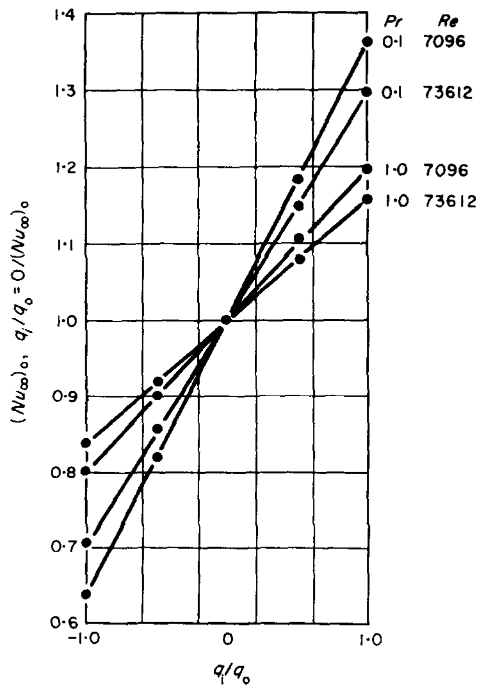

FIG. 11. Effect on unsymmetrical heat fluxes on fully developed Nusselt numbers.

(Points represent solutions by Hatton and Quarmby).

THE LINEARITY OF $\dfrac{Nu_{o,\ q_i/q_o = 0}}{Nu_o}$

This ratio is easily obtained from equation (25) by substituting $q_i/q_o = 0$, and dividing by Nu_o

$$\frac{Nu_{o,\ q_i/q_o = 0}}{Nu_o} = 1 - \frac{G_o/G_i \sum_1^\infty C_n(-1)^n \exp(-8\lambda_n^2 x^+/Re)}{1 - \sum_1^\infty C_n \exp(-8\lambda_n^2 x^+/Re)} \left(\frac{q_i}{q_o}\right) \quad (26)$$

That is, with uniform heat input, and given Reynolds and Prandtl numbers, $1/Nu_o$ is linear with respect to q_i/q_o. For fully developed temperature profiles equation (26) reduces to:

$$\frac{(Nu_\infty)_{o,\ q_i/q_o = 0}}{(Nu_\infty)_o} = 1 - (G_o/G_i)\frac{q_i}{q_o} \quad (27)$$

Equations analogous to (26) and (27) can be obtained from equation (24); but with the suffixes o and i on Nu

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and q interchanged. Equation (26) like equation (27) lacks generality because of the assumption of constant fluid properties.

However, by calculating characteristic heat-transfer coefficient from experimentally observed data may partly compensate for this.

A CHARACTERISTIC HEAT-TRANSFER COEFFICIENT

If a heat-transfer coefficient is defined

$$h_s = (q_i - q_o)/(t_i + t_o - 2t_m)$$

it can be shown from equations (24) and (25) by solving for $t_i - t_m$ and $t_m - t_o$ respectively, adding the equations and simplifying that

$$Nu_s = \frac{1}{[1 - 2\sum_1^\infty C_{2n} \exp(-8\lambda_{2n}^2 x^+/Re)]\, G_i + G_o} \tag{28}$$

or since this equivalent to equations (24) and (25) when $q_i/q_o = -1$

$$Nu_s = Nu_{i(q_i/q_o = -1)} = Nu_{o(q_i/q_o = -1)} \tag{29}$$

and consequently

$$h_s = h_{i(q_i/q_o = -1)} = h_{o(q_i/q_o = -1)} \tag{30}$$

APPLICATIONS

If a number of experiments are performed at the given Reynolds and Prandtl number but with different heat-flux ratios, a change in Nu_s will be caused solely by the change in fluid physical properties.

Using this approach it is theoretically possible to estimate the heat-transfer coefficients for an arbitrary q_i/q_o ratio from data obtained in a single experiment; of course, on the assumption of constant fluid properties. If—during an experiment with uniform heat input and one wall insulated—the temperatures of both walls, heat flux, and fluid mean temperature are measured; it is possible to calculate both $h_{i(q_o/q_i = -0)}$ and $h_{s(q_o/q_i = -1)}$.

Because $1/Nu_i$ is linear with respect to q_o/q_i, heat-transfer coefficients for other symmetries may be estimated.

The above procedures are not restricted to turbulent flow since they depend solely on the superposition of solutions of the differential equation for heat transfer.

DERIVATION OF EQUATION (28)

In the table of nomenclature Hatton and Quarmby define the Nusselt number as hd/k, and the heat-transfer coefficients are $h_i = q_i/(t_i - t_m)$ and $h_o = q_o/(t_m - t_o)$ at the near wall and the far wall respectively. $Nu_i = q_i d/k(t_i - t_m)$ and $Nu_o = q_o/k(t_m - t_o)$ are substituted in equations (24) and (25), and the equations are solved for the respective temperature differences.

$$t_i - t_m = \frac{d}{k}\{q_i G_i [1 - \sum_1^\infty C_n \exp(-8\lambda_n^2 x^+/Re)] - q_o [G_o - G_i \sum_1^\infty C_n(-1)^n \exp(-8\lambda_n^2 x^+/Re)]\} \tag{31}$$

$$t_m - t_o = \frac{d}{k}\{q_o G_i [1 - \sum_1^\infty C_n \exp(-8\lambda_n^2 x^+/Re)] - q_i [G_o - G_i \sum_1^\infty C_n(-1)^n \exp(-8\lambda_n^2 x^+/Re]\} \tag{32}$$

Subtracting equation (31) from equation (30)

$$t_i + t_o - 2t_m = \frac{d}{k}\{(q_i - q_o)\, G_i [1 - \sum_1^\infty C_n \exp(-8\lambda_n^2 x^+/Re)] + (q_i - q_o)[G_o - G_i \times \sum_1^\infty C_n(-1)^n \exp(-8\lambda_n^2 x^+/Re)]\} \tag{33}$$

or separating heat fluxes, temperature differences, mean diameter and fluid thermal conductivity.

$$Nu_s = \frac{h_s d}{k} = \frac{(q_i - q_o)\, d}{(t_i + t_o - 2t_m)\, k} = \frac{1}{G_i [1 - \sum_1^\infty C_n \exp(-8\lambda_n^2 x^+/Re] + G_o - G_i \sum_1^\infty C_n(-1)^n \exp(-8\lambda_n^2 x^+/Re)}$$

since the odd eigenfunctions cancel out we get equation (28). It is evident that the thermal boundary conditions on the left must always satisfy the function on the right, and that this function is constant for a given Reynolds and Prandtl number. All h_s values for a given fluid under constant flow conditions are therefore equal. The odd eigenfunctions also cancel out when $q_i/q_o = -1$ in either equation (24) or (25), the characteristic coefficient is consequently numerically equal to the coefficients for the special case of symmetrical heat fluxes.

REFERENCES

1. Niels Madsen and A. Ralph Thompson, Heat transfer to a fluid flowing between parallel planes, unpublished manuscript.